If δ_1 is contained in $P-R$, then $P=R+\delta_1 n\lambda$, where λ i some integer and $n \overline{>} 1$. Hence,

$$P^{\delta_1}=R^{\delta_1}+\delta_1{}^{n+1}\lambda+\ldots\equiv R^{\delta_1} \text{ (mod. } \delta_1{}^{n+1}).$$

Hence, $P^{\delta_1}-R^{\delta_1}$ is divisible by $\delta_1{}^{n+1}$. But $P^\alpha - R^\alpha$ is divisible by $\delta_1{}^{n+1}$. Now $P^{\delta_1}-R^{\delta_1}$ is the lowest number of this form which is divisible by $\delta_1{}^{n+1}$. Hence δ_1 is a factor of α.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

273. Proposed by THEODORE L. DE LAND, Treasury Department, Washington, D. C.

Three ingots of the precious metals were received at the Mint for assay, where it was found as follows: That in 3 grains of the first ingot and 2 grains of the second the gold was 3 times the silver; that in 2 grains of the first and 6 grains of the third the gold was 8 times the copper; that in 2 grains of the second and 3 grains of the third the silver was 5 times the copper; that in 1 grain of the first, 2 grains of the second, and 3 grains of the third the gold was 2 times the silver; that in 1 grain each of the first and second ingots there were 11 parts of gold to 5 parts of silver; and that 6 grains of the first, 5 grains of the second, and 2 grains of the third on being assayed proved to be 17 carats gold fine. There was no trace of any other metal in the ingots.

Required: The theoretical analysis of each of three ingots.

Solution by the PROPOSER.

Let x=the fraction of gold in a grain of the first ingot;
y=the fraction of silver in a grain of the first ingot;
$1-(x+y)$=the fraction of copper in a grain of the first ingot; and
z=the fraction of gold in a grain of the second ingot;
u=the fraction of silver in a grain of the second ingot;
$1-(z+u)$=the fraction of copper in a grain of the second ingot; and
v=the fraction of gold in a grain of the third ingot;
w=the fraction of silver in a grain of the third ingot;
$1-(v+w)$=the fraction of copper in a grain of the third ingot.

There are six unknown quantities and six conditions in the solution and problem which may be equated as follows:

First,	$3x+2z=3[3y+2u]\ldots(1)$;
Second,	$2x+6v=8\{2[1-(x+y)]+6[1-(v+w)]\}\ldots(2)$;
Third,	$2u+3w=5\{2[1-(z+u)]+3[1-(v+w)]\}\ldots(3)$;
Fourth,	$x+2z+3v=2(y+2u+3w)\ldots(4)$;
Fifth,	$x+z : y+u :: 11 : 5\ldots(5)$; and
Sixth,	$6x+5z+2v : 13 :: 17 : 24\ldots(6)$.

By elimination, we have $x=1$, $y=0$, $1-(x+y)=0$, $z=\frac{3}{8}$, $u=\frac{5}{8}$, $1-(z+u)=0$, $v=\frac{2}{3}$, $w=\frac{5}{24}$, and $1-(v+w)=\frac{1}{8}$.

We interpret these results as follows: That the first ingot was pure gold; that the second ingot was 9-carat gold, and 15-carat silver; and that the third ingot was 16-carat gold, 5-carat silver, and 3-carat copper.

Also solved in the same manner by G. B. M. Zerr, S. A. Corey, L. E. Newcomb, and G. W. Greenwood.

274. Proposed by R. D. CARMICHAEL, Anniston, Ala.

Find the limit of $\frac{3^2+1}{3^2-1}\cdot\frac{5^2+1}{5^2-1}\cdot\frac{7^2+1}{7^3-1}\cdot\frac{11^2+1}{11^2-1}\cdots$ where the squared numbers are the natural odd *primes* in order.

Solution by G. B. M. ZERR, Ph. D., Parsons, W. Va., and J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Putting the expression in the form

$$\frac{(1+1/3^2)(1+1/5^2)(1+1/7^2)\ldots}{(1-1/3^2)(1-1/5^2)(1-1/7^2)\ldots}=\frac{s}{s^1}$$

and remembering that $(1+1/2^2)s=\frac{15}{\pi^2}$ and $(1-1/2^2)s_1=\frac{6}{\pi^2}$, (pp. 133-134, Vol. V, No. 5), we have $s/s_1=1\frac{1}{2}$.

C. N. Schmall gives the following arithmetical solution of 269. When the boats first meet, combined distance traveled is equal to width of river; when they meet for the second time the distance traveled is equal to three times the width of river and that each boat has gone three times as far as when they first met. Hence one has gone 3×720 yards$=$ 2160 yards and has made one trip and 400 yards of the return trip· Hence, width of river $=$2160 yards—400 yards$=$1760 yards$=$1 mile.

GEOMETRY.

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302. Proposed by F. H. SAFFORD, Ph. D., University of Pennsylvania.

Through a given point within a circle draw any two chords, also a radius and a secant perpendicular to the radius. Let the extremities of the chords be taken as the vertices of a quadrilateral. Show that the sides of the quadrilateral, produced when necessary, cut the secant in points equidistant, in pairs, from the given point. [A proof by Euclidean geometry is preferred, as the problem was originally given to a high school class.] Must the given points be within the circle?